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METHODOLOGY FOR DETERMINING
SAMPLING INTERVALS

H.J. Larson and T. Jayachandran

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METHODOLOGY FOR DETERMINING SAMPLING INTERVALS

by

H.J. Larson and T. Jayachandran

ABSTRACT

A new methodology for the determination of sampling intervals to be used with the spectrometric oil analysis program have been developed. The methodology has been tested with preliminary data and appears to perform well.

1. Introduction

Several documents have been received which present ideas for determining the "optimum" sampling interval for various types of equipment (see [2], [3], and [7]). Both [3] and [7] suggest the use of methodology described by Hudson in [1] to fit a segmented-line to the iron readings observed before the occurrence of a JOAP hit. The join point (in flight hours before the hit) of the two line segments for a given piece of equipment, is then used to establish the sampling interval for the equipment concerned.

Since none of these documents describes an actual physical model in any detail, which would lead to the procedures employed, we shall first attempt to specify such a model and describe the procedures already suggested. We shall the apply the segmented-line methodology to some recently collected data, followed by some comments about this methodology. Finally, we shall suggest what we feel to be a better approach and illustrate this suggested approach with some data collected by Luke AFB. Recommendations and conclusions are presented in the final section.

2. A Simple Model

The lubricating fluid in any piece of equipment is meant to reduce friction and reduce wear of moving parts. In so doing the fluid itself may become contaminated by the metals it is lubricating. It seems reasonable to assume that the level of contamination of the fluid is a nondecreasing function of time, so long as no new fluid is added to the system. Thus, if we think of the level of contamination of the fluid by iron, say, as a function of time, it is possible that it would look something like Figure 1. The vertical axis depicts the level of iron contamination and the horizontal (real clock) time. When the equipment is not in operation (between flights) it seems reasonable that

the level of iron contamination does not change (again assuming no new fluid is added). During operation of the equipment, the level of contamination may change and, if so, the level can only increase. Thus, in Figure 1 the contamination level is increasing during each of the four flights. It is, of course, not necessary that the increase be linear as pictured for flights 1, 2, and 4; a linear rate of increase gives the simplest possible model, is the easiest to work with, and may give a sufficiently accurate approximation to prove useful. Note that in Figure 1, the rate of increase shifts during flight 3.

If we do make the assumption that the level of contamination does not change during nonoperating hours, as pictured in Figure 1, we might just as well ignore these periods and consider the level of contamination as a function of flight time, rather than real time, as pictured in Figure 2. In so doing, the picture becomes quite simple, two intersecting straight lines with the intersection occurring at time T. If the actual iron content were as pictured in Figure 2, it would appear that some event had occurred at time T (actually during Flight 3) causing the rate of buildup to suddenly become greater. This type of occurrence may have implications regarding the time of failure for the equipment.

To attempt to monitor the level of iron contamination in the lubricating fluid (and the level of several other contaminants) a small sample of the fluid can be removed periodically and analyzed on a spectrometer. The spectrometer will produce a set of numbers which are estimates of the concentration levels of the contaminants being monitored at the time the sample was taken. Thus, the sequence of iron measurements, for example, can be used to estimate the level of iron contamination as a function of flight time.

It seems plausible that a normally functioning engine may have some (relatively low) rate at which iron, and the other metals which contact the lubricating fluid, will accumulate in the fluid; this might be as pictured for

Contaminant Level (PPM)
Flight 1
Flight 2
Flight 3
Flight 4

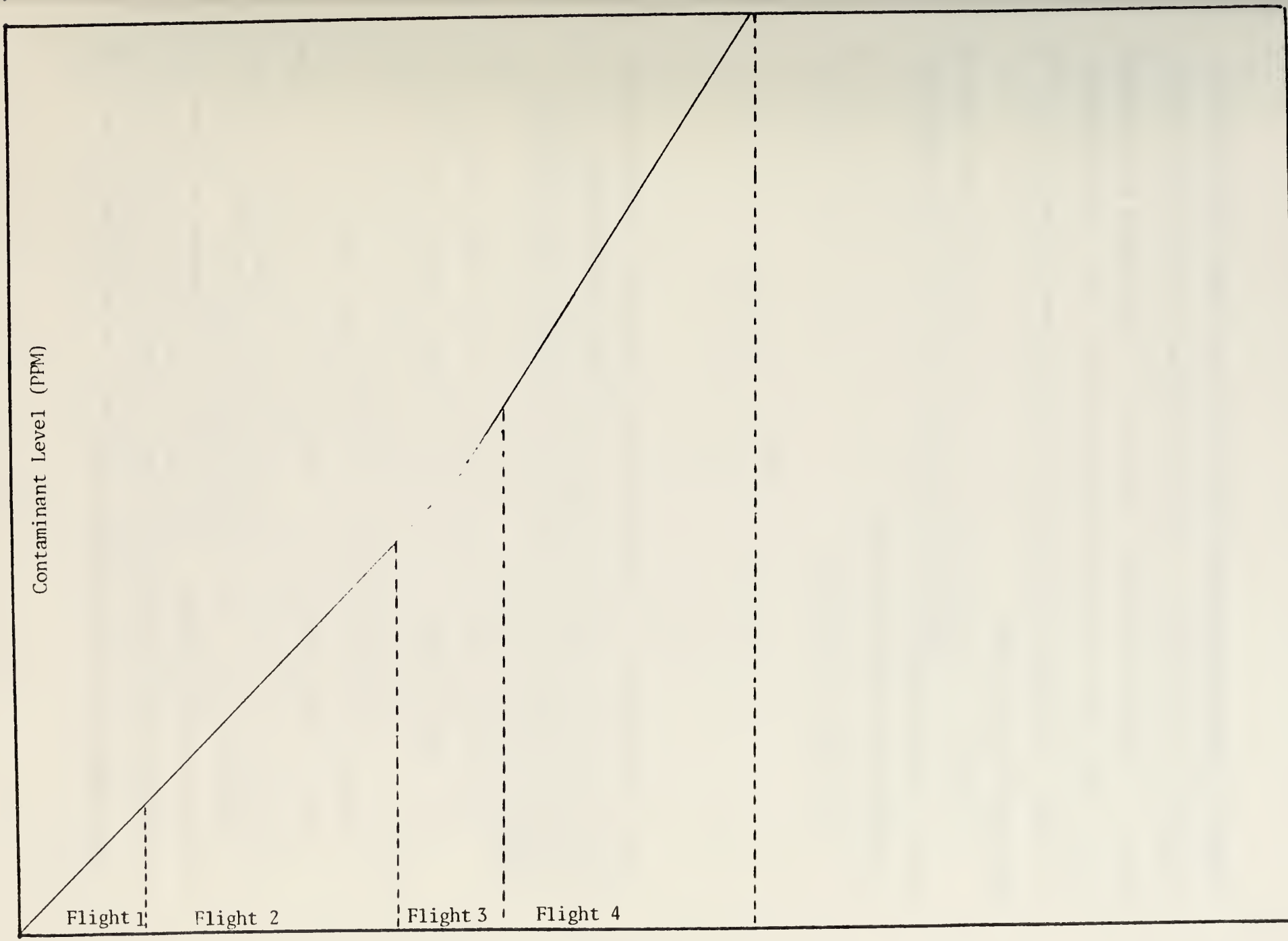

Figure 2

flights 1, 2 and the early part of flight 3 in Figure 1 and, equivalently, for flight time $t \leq T$ in Figure 2. Any rate of increase in excess of this normal (for $t > T$ in Figure 2) then may be indicative of some excessive wear occurring which, in turn, calls for special maintenance action. It is the establishment of "normal" wear rates, and the detection of any increase in rate, for which the JOAP samples may be useful.

To summarize, a model which seems to be tacitly behind the reports which are listed in the bibliography can be described as follows (for each of the metals contacted by the lubricating fluid).

"True" level of contamination

$$y = a_1 + b_1 t \quad , \quad \text{for } t \leq T$$

$$= a_2 + b_2 t \quad , \quad \text{for } t > T$$

where t is the number of flight hours since the lubricating fluid was changed. Granted the fluid is sampled and analyzed on a spectrometer at times $t_1, t_2, \ldots t_n$, the "true" contamination levels are $y_1, y_2, \ldots y_n$ and the (iron) spectrometer readings are $Y_1, Y_2, \ldots, Y_n$ then

$$Y_i = y_i + e_i$$

$$= a_1 + b_1 t_i + e_i \qquad \text{for } t_i \leq T$$

$$= a_2 + b_2 t_i + e_i \qquad \text{for } t_i > T \ ;$$

the quantities $e_i = Y_i - y_i$ are the differences between the "true" content and the spectrometer reading. These are errors of measurement and assumed to be independent random variables with mean 0, variance σ^2; it is quite likely that a normal distribution provides a good model for the probability law of the e_i's.

The methodology described by Hudson [1], and utilized by [3] and [7] uses the observed spectrometer readings to estimate a_1, b_1, a_2, b_2 and T. This is accomplished by minimizing

$$\sum_{i=1}^{m} (Y_i - \hat{a}_1 - \hat{b}_1 t_i)^2 + \sum_{i=m+1}^{n} (Y_i - \hat{a}_2 - \hat{b}_2 t_i)^2$$

with respect to $\hat{a}_1$, $\hat{b}_1$, $\hat{a}_2$, $\hat{b}_2$, subject to the constraints $\hat{a}_1 + \hat{b}_1 T = \hat{a}_2 + \hat{b}_2 T$, $t_m \leq T \leq t_{m+1}$, over all possible values for m. The estimate for T then is given by

$$T = \frac{\hat{a}_1 - \hat{a}_2}{\hat{b}_2 - \hat{b}_1}$$

3. Previously Suggested Methodology

As already mentioned, both references [3] and [7] use Hudson's methodology described in [1], in essentially the same way. For any given equipment model, both references suggest that the estimated segmented line be the basis for determining the "optimum" sampling interval for that equipment model. Both references suggest the isolation of JOAP hits, labelling the time of the hit as t = 0; the oil analysis records prior to the occurrence of the hit then are measured in units before the hit, using $t < 0$. Neither reference explicitly discusses the differences or similarities between different serial numbers of the same model. They tacitly assume the behavior to be the same for all serial numbers and combine data over serial numbers in computing $\hat{T}$. If the different serial numbers (of the same equipment model) do not behave exactly the same (in terms of both rate of accumulation of contaminants and in

terms of beginning level of contamination), then the differences between individual serial numbers adds to the "noise" or measurement error, making estimates of the a_i's, b_i's and T less precise.

First let us discuss the methodology suggested by [7] for determining the optimum sampling interval. They use the estimated value for T directly to determine the optimum sampling interval and recommend that this interval be T/2 for single-engine aircraft, T for twin-engine aircraft and 3T/2 for multiengine aircraft and auxiliary equipments. This reference furthermore states that these sampling intervals will give, respectively,

a) 100 percent probability of obtaining two samples during the abnormal wear period for single-engine aircraft,

b) 100 percent probability of obtaining at least one sample within the abnormal wear period,

c) 67 percent probability of obtaining one sample during the abnormal wear period for the final category.

Although not explained in the reference, these probability statements are undoubtedly based on an underlying uniform probability assumption of the following kind. If, in fact, samples are taken every T/2 flight hours, then any interval of length T, no matter where it starts, must contain (at least) two sampling times. If samples are taken every T hours, every interval of length T must contain at least one sampling time and, if they are taken every 3T/2 hours, then 2/3 of the intervals of length T (depending on where they start) will contain one sampling time and the other 1/3 of the intervals will not. Since T is not known and could be, at best, only estimated from operational data, the relevancy of these statements is not obvious.

Reference [3] also fits segmented lines to observed data with the origin determined as mentioned earlier, combining all serial numbers together. This reference, however, does not directly use the estimate for T in defining the sampling interval. Rather, it assumes the estimated segmented line represents the actual true situation for the amount of contaminant in the fluid, for all serial numbers of the same model, and then sets ppm contaminant guidelines for the model by the height of the segmented line. For example, the ppm content one "average" flight time before the hit is used to set the "T" code value, 1 ppm below this level is the upper "K" code value, etc.

4. An application of the Segmented-Line Methodology

Recent (December 1979) data collected by Kelly AFB has been examined for suitability in applying the segmented-line methodology. In order to apply the methodology it is necessary that

(a) a hit was scored (to establish the zero point)

(b) at least four complete sampling records prior to the hit be available (to allow estimation of the lines)

(c) the data used is complete with no obvious errors or inconsistencies

Application of (a), (b), (c) did not leave many equipments from which to choose, in using the December 1979 data. Nine T-38A aircraft (J85-5 engines) were found to satisfy (a), (b), (c). The segmented-line methodology was applied to each of these aircraft separately; then the records for all nine aircraft were combined and a composite segmented-line was computed. This enables one to see the "best" segmented line for each individual aircraft, as well as how these compare with the "best" composite segmented line. Table 1 gives the results of these computations, the values for a_1, a_2, b_1, b_2 and T for the ten cases. The ten segmented lines are plotted in Figure 3.

AIRCRAFT TYPE: T-38A ENGINE: J85-5

SERIAL NUMBER	a_1	b_1	a_2	b_2	JOINT POINT T	TYPE OF FAILURE
230583(1)	39.60	-.192	52.1	.689	- 14.2	Reduction Gear Bearing
231226(2)	8.76	.0269	32.0	1.15	- 20.7	Reduction Gear Bearing
231588(3)	22.0	.079	14.3	0.0118	-118.0	Reduction Gear Bearing
232510(4)	40.5	.668	51.3	0.813	- 14.4	#2 Bearing Housing
232214(5)	44.6	.122	57.0	2.6	- 5	#2 Bearing Housing
232688(6)	3.67	.0354	18.5	4.5	- 3.32	Aux Drive Gearbox Transfer
230547(7)	24.2	.0202	34.0	1.29	- 7.73	Aux Dr Gearbox Transfer
230534(8)	3.21	-.0107	9.0	.643	- 8.85	Rear Accessory Drive Shaft
231208(9)	70.2	.345	44.3	.0472	- 87.2	Aux Drive-Accessory Oil Pump
Composite	22.8	.0168	35.9	6.55	- 2.0	Pooled Over All JOAP Hits

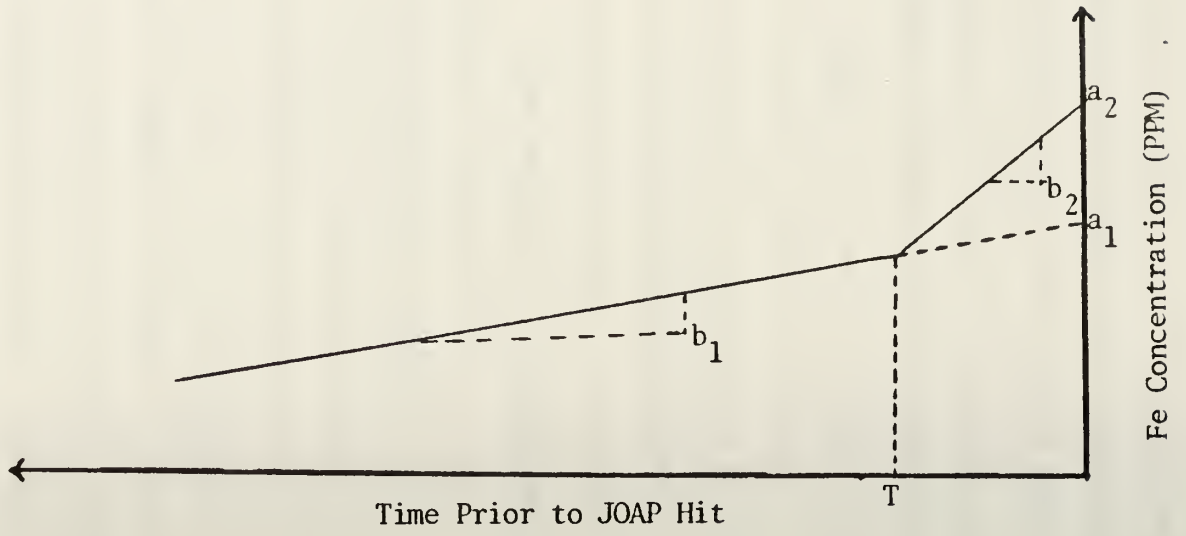

TABLE 1

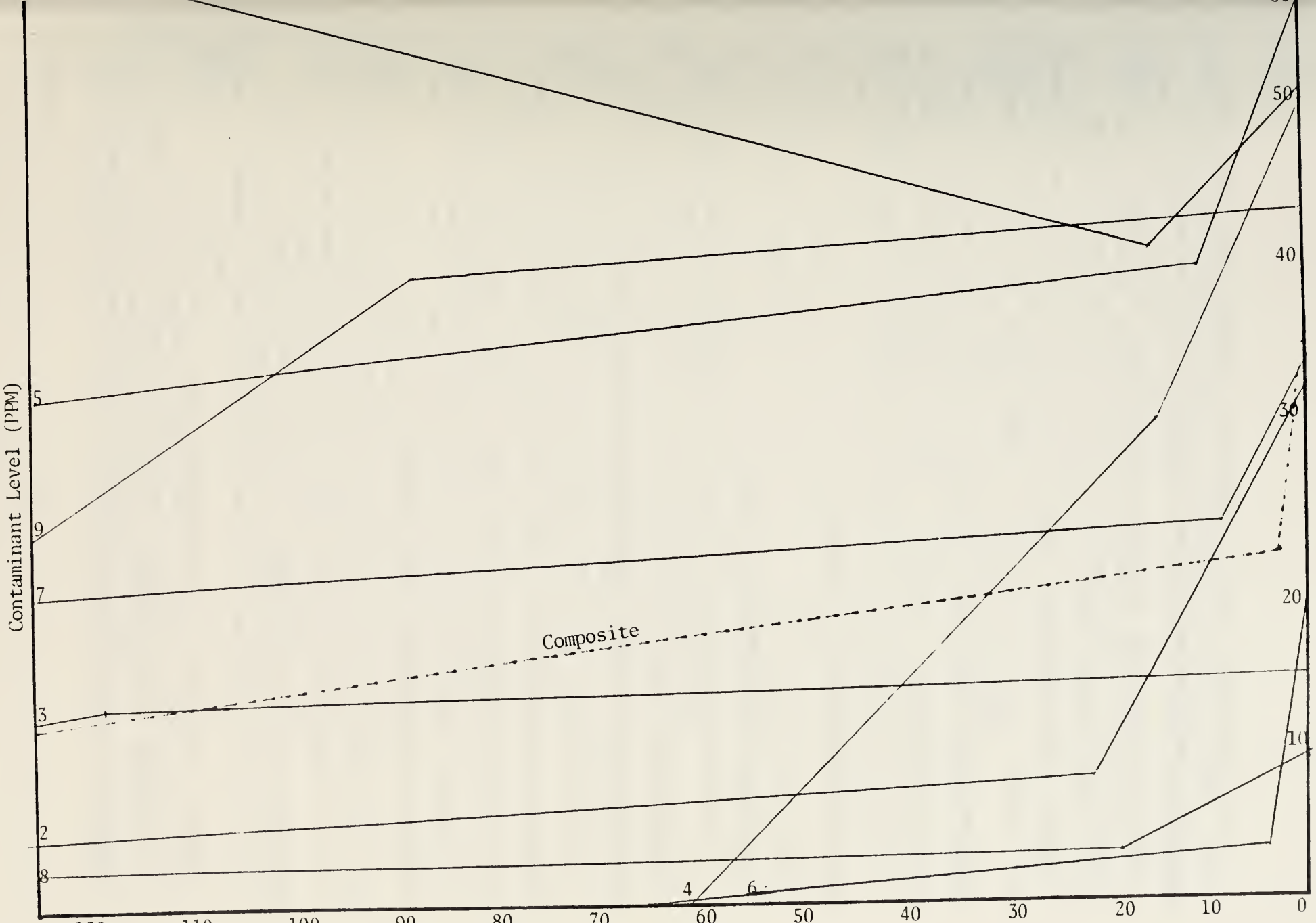
Contaminant Level (PPM)
50
40
30
20
10
5
9
7
3
2
8
4
6
Composite
120
110
100
90
80
70
60
50
40
30
20
10
0
Time Prior to JOAP Hit

Figure 3

It is interesting to note the wide disparity between these lines, even among those serial numbers which were diagnosed to have the same type of failure. This would seem to indicate that individual serial numbers may present different behavior, even when they are suffering the same type of failure mechanism. Combining all nine together mixes not only different serial numbers but different failure modes as well. It seems doubtful to us that this composite segmented line provides a useful description for any single one of the aircraft whose records were used to derive it. Using the composite line to set sampling intervals for all aircraft of this type or to determine T, K, etc., code values, does not seem particularly appealing.

5. Critique of the Segmented-Line Model

The methodology described by Hudson [1] is designed for cases in which the underlying phenomenon (contamination buildup) occurs at two different rates. His procedures then are useful in estimating the two rates and the time T at which the rate shifted. It may be that some equipment failure modes exhibit this type of behavior. It seems likely, though, that many, if not most, failure modes may go through a gradual, continuous change in rate, rather than an abrupt shift in rate at a specific time T. Such failure modes would not have a well-defined, easily detectable time T associated with them.

If a particular failure mode does have a time T at which the rate of accumulation abruptly shifts, it is not obvious that exactly the same value of T is appropriate for all serial numbers of the same model (see Figure 3). This would imply that individual serial numbers should be treated separately in general. A value of T would have to be estimated for each. If each of, say, n different serial numbers of the same model, are combined together to get a composite estimate for T, any differences between the T values for the different serial

numbers adds to the observation error in estimating the a_i's, b_i's and T. Even if serial numbers are treated individually in estimating T, after a hit has been scored and the offending part(s) repaired or replaced, the repaired equipment may not have the same value for T as before. This would make the length of the sampling interval determined from some preceding estimate of T, for this serial number, rather suspect for future use with the same serial number.

Several different contaminants are monitored simultaneously with the spectrometer readings. If an estimated T value is derived, for a given serial number, for each of nickel, silver, and titanium, say, it is not obvious what may be the best way of using these separate T estimates to derive a single T to be used in the procedures suggested by either [3] or [7]. Along similar lines, neither reference specifically addresses the question of how many sample values, prior to the hit at time t = 0, should be used in the estimation of T.

Perhaps most importantly, even if we assume the existence of a rate shift time T, and know how to combine the different elements and how many sample values to use, employing operational data to estimate T is still suspect. If the rate of accumulation does shift dramatically at a time T, a JOAP recommended teardown should score a hit at any time after the shift occurs. Using operational data to measure backward in time before the hit was scored really just estimates the time from the shift (assuming one occurred) until the hit was made. It does not estimate the shortest possible time from the shift in rate until a hit could be (or could have been) made, which is really the point of interest.

We feel that any operational procedure to determine sampling intervals should possess two attributes:

(a) It should be capable of recognizing the trace or signature of metallic contamination which is normal for a correctly operating piece of equipment; this signature may likely be unique for individual serial numbers of the same equipment.

(b) It should be capable of detecting changes in the signature which in turn may indicate abnormal wear or some condition which should be corrected.

In the following section we shall describe a procedure which has these attributes and illustrate the computations involved using some F100 data.

6. An Alternative Procedure

We shall describe a procedure which should work well for any equipment whose lubricating fluid is changed on a routine basis. Let A represent the number of flight hours at which the maintenance manual says the fluid should be changed. As already discussed, it seems plausible that an engine in proper working order will have some tendency to accumulate metallic contaminants, in its lubricating fluid, as the flight hours go from 0 to A; the metals which are in contact with the fluid are the ones which one would expect to find comtaminating the lubricating fluid. Whatever the pattern of accumulation may be as t (the number of flight hours) varies from 0 to A, it seems reasonable that this pattern should at least roughly repeat itself after the fluid is changed, so long as the equipment is in proper working order. The suggested procedure, then, is to use operational data, for a given serial number, to estimate what this pattern is as t varies from 0 to A, while the equipment is in proper working order; once this pattern has been estimated the spectrometer results for any sample received from this equipment (taken, say, t* hours after the oil was changed) are compared with what one expects based on the normal pattern. If the spectrometer readings at t* hours are sufficiently out of line with the normal expected readings, then sampling may be done more frequently, or the equipment may be grounded, etc. The rules for doing this are spelled out below.

Let k represent the number of different metals in contact with the lubricating fluid; k may be determined from the engine troubleshooting guide in the JOAP

laboratory manual. These k metals, then, should be the ones which may accumulate in the lubricating fluid as the equipment is used; silicon should additionally be monitored to observe possible dirt accumulation in the fluid. While the equipment is known to be in good working order, the lubricating fluid is sampled after every flight, for at least one cycle of fluid changes. For example, suppose the fluid is to be changed every A = 100 flight hours. The new fluid placed in the engine is analyzed on the spectrometer at $t_1 = 0$ flight hours. Then, after the first flight of duration t_2 hours, the fluid is analyzed again, as it is after the second flight (t_3 total hours since oil change), etc. A final sample is taken and analyzed after the last flight before the fluid is changed; the accumulated number of hours for this final sample is t_n, which then is roughly equal to A, the maintenance manual recommended length of time for fluid change.

Thus, at times $t_1 = 0, t_2, t_3, \ldots, t_n \doteq A$ we have readings from the spectrometer to use in estimating the "normal" trace of the equipment in good repair. Let $Y_1(t_1), Y_2(t_1), \ldots, Y_k(t_1)$ represent the spectrometer readings for the k elements sampled, at time $t_1 = 0$; these are the contamination levels of the new fluid put into the aircraft. Similarly, let $Y_1(t_i), Y_2(t_i), \ldots, Y_k(t_i)$ represent the k spectrometer readings at time t_i, $i = 2,3,\ldots n$. We shall assume that

$$Y_j(t_i) = \beta_{j0} + \beta_{j1} t_i + e_{ji} \quad ,$$

that is, each of the k elements may change linearly in concentration with flight time; β_{j1} represents the rate of change in concentration for element j and β_{j0} represents the initial concentration of element j in the fluid. The quantities e_{ji} are observation or measurement errors, caused by inhomogeneities of the fluid,

the inherent randomness of the spectrometer readings, etc. It seems reasonable to assume that, for each element, these e_{ji}'s are independent, normal, each with variance σ_j^2, at least for reasonable amounts of change in the level of concentration.

Standard statistical theory (see the appendix) can be used to estimate the values for $\beta_{j0}, \beta_{j1}, \sigma_j^2, j = 1,2,\ldots,k$, given the n readings for each element over one or more cycles. These estimates are denoted by b_{j0}, b_{j1}, s_j^2, respectively. Rather than setting limits for the k elements individually, it seems practical to find the single linear combination, $Y(t) = \Sigma\, c_j\, Y_j(t)$, which is most informative, in a sense, regarding rates of accumulation of the contaminants and regarding changes in rates.

For any set of constant weights, $c_1, c_2, \ldots, c_k$, our assumed model implies

$$\begin{aligned} Y(t_i) &= \Sigma\, c_j\, Y_j(t_i) \\ &= \Sigma\, c_j(\beta_{j0} + \beta_{j1}\, t_i + e_{ij}) \\ &= \Sigma\, c_j \beta_{j0} + t_i\, \Sigma\, c_j\, \beta_{j1} + \Sigma\, c_j\, e_{ij} \quad ; \end{aligned}$$

that is, we would expect $Y(t)$ to also be a linear function of the number of flight hours. If an oil sample is received, taken t hours after the oil was changed, the value we would expect for $Y(t)$, based on the original sample data from this aircraft then is $\hat{Y}(t) = \Sigma\, c_j\, b_{j0} + t\, \Sigma\, c_j\, b_{j1}$. The estimated signal-to-noise ratio, frequently employed in many technical areas, is, for a sample taken t hours after the oil was changed,

$$\frac{\hat{Y}(t)}{\sqrt{\text{Var}\ \hat{Y}(t)}} = \frac{\Sigma c_j\, b_{j0} + t\, \Sigma\, c_j\, b_{j1}}{\sqrt{\text{Var}\ \hat{Y}(t)}} \quad ;$$

choosing the c_j's to maximize this quantity gives the "most sensitive" linear combination Y(t), in some senses. It can be shown (see the appendix) that the c_j values which accomplish this are

$$c_j = \frac{b_{j0} + b_{j1}\, t}{\sqrt{\text{Var } \hat{Y}_j(t)}}$$

when the readings of the individual elements are assumed independent of each other.

To keep the resulting range of Y(t) values on roughly the same scale as the original element readings, it is convenient to normalize these weights so they add to one; this is done by simply dividing each c_j by the total of all c_j's. It will be noted that these normalized c_j's depend on t; the weights which maximize the signal-to-noise ratio thus would change as t changes. Rather than requiring a new computation of weights for every sample received, we propose a single set of weights be computed for a single value of t determined by the following reasoning.

As mentioned above

$$Y(t_i) = \Sigma\, c_j\, \beta_{j0} + t\, \Sigma\, c_j\, \beta_{j1} + \Sigma\, c_j\, e_{ij}$$

and, if the individual element readings are taken to be independent, $\text{Var}[Y(t_i)] = \Sigma\, c_j^2\, \sigma_j^2$ for each t, which is estimated by $\Sigma\, c_j^2\, s_j^2$. It would seem rational to additionally require that the c_j's be chosen to minimize this estimated variance, $\Sigma\, c_j^2\, s_j^2$. The values which accomplish this cannot be expressed in closed form, but can be numerically determined from an equation discussed in the appendix. Thus the weights $c_1, c_2, \ldots, c_k$ are chosen so that (a) they maximize the estimated signal-to-noise ratio, (b) they are normalized

to add to one, (c) they minimize the estimated variance of the composite $Y(t) = \Sigma\, c_j\, Y_j(t)$, for $0 < t \leq A$.

Granted the weights, $c_1, c_2, \ldots, c_k$ have been determined for a given serial number, they can be used to determine bounds which should not be exceeded (with a given probability) for the composite reading taken at any number t, of hours after the oil has been changed. The details of accomplishing this are discussed in the appendix. Here we shall simply discuss the use of such bounds for the oil analysis program.

For any given serial number, known to be in good working order, oil samples are analyzed after each flight through at least one cycle of routine oil changes, giving n sets of contaminant readings. For those metals in contact with the oil, the estimates b_{j0}, b_{j1}, s_j^2 , are computed, as are the weights, $c_1, c_2, \ldots, c_k$. At each oil change, the new oil put into the equipment is analyzed and then routine samples are collected at the normal rate for the equipment (perhaps every 20 or 30 flight hours, depending on the equipment). For a routine sample taken t hours after the oil was changed, the bounds $y_{.9}(t)$ and $y_{.99}(t)$ are computed (these bounds depend on t and are values which would be exceeded only 10 percent and 1 percent of the time, respectively, if the equipment is in good order). The sample is then analyzed on the spectrometer and the composite sum, $Y(t) = \Sigma\, c_j\, Y_j(t)$, is computed and compared with the bounds. If $Y(t) < y_{.9}(t)$, it appears the equipment is in good order so sampling continues at a routine rate. If $y_{.9}(t) \leq Y(t) < y_{.99}(t)$, either an unusual reading has occurred (if the equipment is in good order), or the equipment may no longer be accumulating contaminants at the same rate so one might want to monitor the accumulation more closely, say sample after every flight. If $Y(t) \geq y_{.99}(t)$, a very unusual set of readings has occurred or the rate of

accumulation has definitely increased a great deal; ground the aircraft and determine the trouble.

In this discussion two bounds, $y_{.9}(t)$ and $y_{.99}(t)$, have arbitrarily been chosen. It is very easy to compute bounds with any given probability value (see the appendix); the probability that $Y(t)$ exceeds $y_p(t)$ is $1 - p$ if the equipment is in good working order. Thus, changing p will change the proportion of the time the bound will be exceeded while the equipment is in good order; p can be set at any desired level. More than two bounds can be evaluated if one wants a larger number of possible actions after each sample. Of course, if grounding is recommended and diagnostic judgements are to be made about the possible trouble, human evaluators referring to all the recent records are to be employed.

The following section discusses the application of this reasoning to some data supplied by Luke AFB. While it has not proved possible to receive records for one or more serial numbers sampled after every flight for at least one cycle of oil changes, the partial-cycle records can still be used to illustrate the methodology suggested.

7. A Numerical Example

We shall illustrate the suggested methodology with data for an F-100 engine, serial number 680123. The data available are 28 sampling records (Table 2) apparently taken after roughly every flight, spanning the interval from $t_1 = 1$ to $t_{28} = 58$ hours after the oil was changed. (But notice the big gap from 9 to 23 hours.) Of the data sets received, for five different serial numbers, this collection is the most complete for what appears to be an engine in good working order. The records stop at 58 hours; we have no record of what

subsequent sampling may have revealed. We shall assume A = 60 hours is the recommended time for oil change, simply to illustrate the suggested methodology. It is quite apparent from the full set of records that A is actually much larger than 60.

From the JOAP troubleshooting diagram, there are six metallic elements in contact with the lubricating fluid; these are fe, ag, al, cr, ni and ti. For all 28 samples available the aluminum reading was 0; because of this, we shall for the example, use only the remaining k = 5 elements. The 28 records used are given in Table 2.

We assume the records in Table 1 to be from this serial number after every flight (again note the gap in t values) while the engine is in proper working order. It is found that the optimal weights (computed with t = A = 60) the c_j values, are to 3-decimals .600, .115, .100, .120, .064 so the composite, "most sensitive" reading is

$$Y(t) = .600Y_1(t) + .115Y_2(t) + .100Y_3(t) + .120Y_4(t) + .064Y_5(t)$$

where $Y_1(t)$ = fe reading, $Y_2(t)$ = ag reading, $Y_3(t)$ = cr reading, $Y_4(t)$ = ni reading, $Y_5(t)$ = ti reading, all at time t. The 90 percent and 99 percent readings have been chosen arbitrarily, and evaluated for samples received t = 10,20,30,40, 50,60 hours after the oil was changed (and assuming the initial contamination of new oil put in the engine at each change is the same as it was for the records in Table 2). These are given in Table 3.

Thus, if a sample is received for this serial number, taken t = 30 hours after the oil was changed, and yields $y_1 = 1$, $y_2 = 0$, $y_3 = 0$, $y_4 = 2$, $y_5 = 1$, the composite reading is y = .6(1) + .115(0) + .1(0) + .120(2) + .064(1) = .904; since for t = 30 this is less than $y_{.9} = 1.938$ (from Table 3, 30 hours) it would

TABLE 2

Sample Records for S/N 680123

t_i	fe	ag	cr	ni	ti
1	1	0	0	0	1
3	1	0	0	0	0
4	1	0	0	0	1
5	1	0	0	0	1
7	2	0	0	0	1
9	2	0	0	0	1
23	2	0	0	0	0
25	2	0	0	0	0
25	2	0	0	0	0
27	2	0	0	0	0
31	2	0	0	0	0
33	2	0	0	0	0
33	2	0	0	0	0
35	2	0	0	0	0
37	2	0	0	0	0
40	2	0	0	0	0
41	3	0	0	0	0
42	4	0	0	1	0
45	3	0	0	0	1
46	4	0	0	1	1
46	3	0	0	0	1
49	3	0	0	1	1
53	3	0	0	0	1
53	3	0	1	1	1
55	3	0	0	0	1
55	3	0	0	0	0
57	3	1	0	1	1
58	3	0	0	0	0

TABLE 3

90% and 99% Limits for S/N 680123

t

	10	20	30	40	50	60
90%	1.469	1.700	1.938	2.181	2.431	2.686
99%	1.970	2.192	2.425	2.669	2.924	3.19

appear the aircraft is behaving the same as during its normal operating phase so sampling would continue at its normal rate. If a sample taken at $t = 50$ hours yielded $y_1 = 4$, $y_2 = 1$, $y_3 = 1$, $y_4 = 2$, $y_5 = 1$, the composite reading is $y = .6(4) + .115(1) + .1(1) + .12(2) + .064(1) = 2.919$ which lies between $y_{.9} = 2.431$ and $y_{.99} = 2.924$; implementing these limits as discussed earlier the recommendation would be to sample at a more frequent rate.

8. Conclusions and Recommendations

Based on the preceding discussion we conclude the following:

1. Many failure modes may not exhibit an abrupt shift in rate of accumulation; it is possible the time T used in the segmented line methodology is not well-defined.

2. Even if the time T is well-defined, it is not apparent that the same T is appropriate for all serial numbers of the same model, nor for all failure modes for the same serial number, nor indeed is it necessarily the same for repeated instances of the same failure mode for the same serial number.

3. If the time T is well-defined, it cannot be accurately estimated by using actual operational data; the time from the shift in rate until a hit is scored is not really of great interest.

4. Any procedure adopted should be one which allows estimation of the normal signature of contaminant accumulation, for a given serial number, and which has the ability to recognize departures from this signature which may indicate excessive wear.

We recommend that the alternative procedure described in section 6 be investigated further and that it be tried for at least five different serial numbers of at least two different equipment models through at least two oil change cycles. This may entail a specially monitored program to gather the appropriate data for the implementation of the methodology.

APPENDIX

1. Estimation of Parameters

The spectrometer readings have been assumed to satisfy the linear model

$$Y_j(t_i) = \beta_{j0} + \beta_{j1} t_i + e_{ji} \qquad \begin{matrix} j = 1,2,\ldots,k \\ i = 1,2,\ldots,n \end{matrix}$$

where

j	is the running subscript to represent the k wear metals included for the analysis
i	is the subscript to represent the n spectrometer readings between oil changes
$Y_j(t_i)$	the i^{th} spectrometer reading on the j^{th} wear metal recorded at time t_i
β_{j0}	the initial concentration level of the j^{th} wear metal in the lubricating fluid
β_{j1}	the rate of change in the concentration level for wear metal j and
e_{ji}	the measurement error corresponding to the i^{th} measurement on the j^{th} wear metal assumed to be normally distributed with mean 0 and variance σ_j^2 .

From standard statistical theory the parameters β_{j0}, β_{j1} and σ_j^2 can be estimated from the data $Y_j(t_i)$ as follows:

Let $S_1 = \sum_{i=1}^{n} t_i$ be the sum of the times (after the last oil change) at which oil samples were taken and analyzed

$S_2 = \sum_{i=1}^{n} t_i^2$ the sum of the squares of times

$S_3(j) = \sum_{i=1}^{n} Y_j(t_i)$ the sum of the n spectrometer readings on element j

$S_4(j) = \sum_{i=1}^{n} Y_j^2(t_i)$ the sum of the squares of the n spectrometer readings on element j

$S_5(j) = \sum_{i=1}^{n} t_i\, Y_j(t_i)$ the sum of the products of the spectrometer readings and the corresponding measurement times

$$S_{tt} = nS_2 - S_1^2$$

$$S_{yy}(j) = nS_4(j) - S_3^2(j)$$

$$S_{ty}(j) = nS_5(j) - S_1 S_3(j) \ .$$

Then, the statistical estimators of β_{j1}, β_{j0} and σ_j^2 are respectively.

$$\hat{\beta}_{j1} = b_{j1} = \frac{S_{ty}(j)}{S_{tt}(j)}$$

$$\hat{\beta}_{j0} = b_{j0} = \frac{1}{n}\,[S_3(j) - b_{j1}\, S_1] \quad \text{and}$$

$$\hat{\sigma}_j^2 = S_j^2 = \frac{S_{tt}\, S_{yy}(j) - S_{ty}^2(j)}{n(n-2)S_{tt}}$$

By way of an illustration, consider the data in Table 2. The times at which oil samples were taken are in column 1 and the spectrometer readings on iron (Fe) the first element considered are in column 2 of the Table. Then n = 28 and

$$S_1 = \sum_{i=1}^{28} t_i = (1+3+4+\ldots+55+57+58) = 938$$

$$S_2 = \sum_{i=1}^{28} t_i^2 = (1^2+3^2+4^2+\ldots+55^2+57^2+58^2) = 40,406$$

$$S_3(1) = \sum_{i=1}^{28} Y_1(t_i) = (1+1+1+\ldots+3+3+3) = 66$$

$$S_4(1) = \sum_{i=1}^{28} Y_1^2(t_i) = (1^2+1^2+1^2+\ldots+3^2+3^2+3^2) = 174.00$$

$$S_5(1) = \sum_{i=1}^{28} t_i\, Y_1(t_i) = (1\times1+3\times1+4\times1+\ldots+55\times3+57\times3+58\times3) = 2551$$

$$S_{tt} = 28\times40,406 - (938)^2 = 1,131,368 - 879,844 = 251,524$$

$$S_{yy}(1) = 28\times174 - (66)^2 = 4872 - 4356 = 516$$

$$S_{ty}(1) = 28\times2551 - (938)(66) = 71428 - 61,908 = 9520$$

$$b_{11} = \frac{9520}{251,524} = .038$$

$$b_{10} = \frac{1}{28}\,[66 - (.038)(938)] = 1.08$$

$$S_1^2 = \frac{251,524\times516 - (9520)^2}{28\times26\times251,524} = .21$$

2. Determination of Optimal Weights for Combining the Estimates on Different Wear Metals

In section 6 it has been suggested that instead of studying the different wear metals included in the analysis separately, it would be preferable to use

an "optimal linear combination" that would maximize the estimated signal-to-noise ratio

$$\frac{\hat{Y}(t)}{\sqrt{\text{Var } \hat{Y}(t)}} = \frac{\sum_{j=1}^{k} c_j(b_{j0} + t\, b_{j1})}{\sqrt{\text{Var } \hat{Y}(t)}}$$

It can be shown that $\text{Var } \hat{Y}(t) = \sum_{j=1}^{k} c_j^2 h_j$ where

$$h_j = \sigma_j^2 \left[\frac{1}{n} + \frac{n(t-\bar{t})}{S_{tt}} \right]$$

and if we let $g_j = (b_{j0} + t\, b_{j1})$ the signal-to-noise ratio to be maximized can written as

$$\sum_{j=1}^{k} c_j g_j \Big/ \left(\sum_{j=1}^{k} c_j^2 h_j \right)^{1/2} .$$

Using standard calculus techniques for maximizing a function the optimal weights c_j must satisfy

$$\frac{g_j}{\left(\Sigma\, c_i^2 h_i\right)^{1/2}} - \frac{c_j h_j \left(\sum_{i=1}^{k} c_i g_i \right)}{\left(\Sigma\, c_i^2 h_i\right)^{3/2}} = 0 ,$$

or equivalently

$$\frac{g_1}{c_1 h_1} = \frac{g_2}{c_2 h_2} = \cdots = \frac{g_k}{c_k h_k} = \frac{\sum_{j=1}^{k} c_j g_j}{\sum_{j=1}^{k} c_j^2 h_j} \quad \text{a constant} .$$

The optimal weights are therefore

$$c_j = \frac{g_j}{h_j} = \frac{b_{j0} + t\, b_{j1}}{\text{Var } \hat{Y}(t)}$$

and the normalized weights are

$$c_j' = \frac{c_j}{\sum_{j=1}^{k} c_j}$$

If it is desired that the weights c_j' satisfy the additional condition that the estimated variance $\text{Var}[Y(t_i)]$ is minimum, then the appropriate weights are obtained by solving simultaneously the following three equations

(1) $$\sum_{j=1}^{k} c_j' = 1$$

(2) $$\frac{g_j}{c_j' h_j} = \text{a constant for all } j$$

and $$\sum_{j=1}^{k} c_j' S_j^2 \frac{dc_j'}{dt} = 0$$

The optimal weights (and t) cannot be obtained explicitly and are determined numerically as the solutions of (1), (2) and

$$\sum_{j=1}^{k} \left(\frac{b_{j1}}{h_j} - 2 \frac{\text{Cov}(b_{j0}, b_{j1}) + t\, \text{Var}(b_{j1})}{h_j} \right) c_j' S_j^2 = 0 \quad ;$$

where $0 < t \leq A$.

3. Determination of Bounds

It is assumed that in any real application of this procedure, the number of data values available will be $n \geq 30$ so standard normal tables would be utilized. If the sample size is smaller than 30, larger quantiles should be chosen (say from Students' t distribution). After the weights c_j' have been determined, the composite estimated line is

$$\hat{Y}(t) = \hat{\gamma}_0 + \hat{\gamma}_1 t$$

where

$$\hat{\gamma}_0 = \Sigma\, c_j' b_{j0} \;, \quad \hat{\gamma}_1 = \Sigma\, c_j' b_{j1} \;.$$

Using standard statistical theory, an upper confidence limit for the weighted spectrometer reading for a sample taken t hours after an oil change is given by

$$y_p(t) = \hat{Y}(t) + z_p s_{\hat{y}} \;,$$

where

$$s_{\hat{y}} = \left[\Sigma\, c_j^2 S_j^2 \left(1 + \frac{nt^2 - 2 S_1 t + S_2}{nS_2 - S_1^2}\right) \right]^{1/2}$$

z_p = 100 p^{th} quantile of the standard normal distribution.

For the earlier illustrations, the values p = .9 and p = .99 were used.

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